5.6

Checkmate!

Solving Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Use multiple representations to model exponential functions.
- Understand the properties of exponent expressions with positive and negative exponents.
- Solve exponential functions graphically and algebraically using common bases and properties of exponents.
- Investigate increasing and decreasing exponential functions.
- Model inequalities in exponential situations.
- Use technology to graph, analyze, and solve exponential functions.

How do scientists measure the intensity of earthquakes? You may know that those scientists who study earthquakes—seismologists—refer to a scale known as a Richter scale when reporting the strength of an earthquake.

The scale generally goes from 1 to 9 (though it doesn't really have an upper limit), but an earthquake which has an intensity of 6 on the Richter scale is 10 times more powerful than an earthquake which measures 5.

One of the strongest earthquakes in history occurred in Chile on May 22, 1960. This earthquake measured an amazing 9.5 on the Richter scale—over 30,000 times stronger than a magnitude 5 earthquake!

PROBLEM 1 Exponential Growth

A famous legend tells the story of the inventor of the game of chess. When the inventor showed the new game to the emperor of India, the emperor was so astonished, he said to the inventor, "Name your reward!"

The wise inventor asked the emperor for 1 grain of rice for the first square of the chessboard, 2 grains for the second square, 4 grains for the third square, 8 grains for the fourth square, and so on.

1. Determine the number of grains of rice on the fifth, sixth, seventh, and eighth squares. Write your answers in the second column of the table.

Square Number	Number of Rice Grains	Power
1	1	
2	2	
3	4	
4	8	
5		
6		
7		
8		
s		

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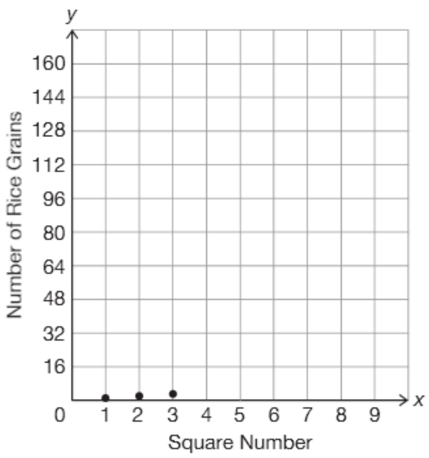
2. What pattern do you notice in the table?

3. A chessboard has 64 squares. Predict how many grains of rice must be on the 64th square if the pattern continues.

4. Graph the points from your table. The first few points have been plotted.



5. Does it make sense to connect the points in this graph? Explain why or why not. If so, connect the points.

6. Complete the third column of the table in Question 1. Write the appropriate power to represent the number of rice grains on the first 8 squares of the chessboard.

7. Analyze your table and write an algebraic expression in the last row to determine the number of rice grains for any square number.

   a. Write an equation in function notation to represent the number of rice grains as a function of the square number, *s*.

   b. Explain how you determined your equation.

8. Pat and George each wrote a function to represent the number of rice grains for any square number using different methods.

Pat

I compared the exponents of the power to the square number in the table. Each exponent is 1 less than the square number.

$f(s) = 2^{s-1}$

George

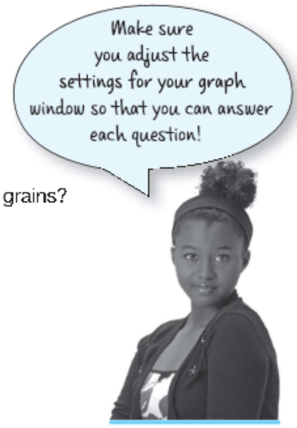
I know this is an exponential function with a common base of 2. If I extend the pattern back on the graph I get the y-intercept of $(0, \frac{1}{2})$, so $a = \frac{1}{2}$.

$f(s) = \frac{1}{2}(2)^s$

Use properties of exponents to verify that 2^{s-1} and $\frac{1}{2}(2)^s$ are equivalent.

9. Use the **intersection** feature of a graphing calculator to answer each question. Write each answer as an equation or compound inequality. Explain how you determined your answer.

a. Which square on the chessboard contains 262,144 rice grains?



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b. Which square on the chessboard contains 32,768 rice grains?

c. Which chess squares on the chessboard contain fewer than 20,000 rice grains?

d. Which squares on the chessboard contain at least 1 million rice grains?

10. Use your calculator to determine the number of rice grains that would be on the very last square of the chessboard.

11. In Question 3, you predicted the number of grains of rice on the last square of the chessboard. How close was your prediction to the actual answer?

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PROBLEM 2 Using Properties of Exponents

An exponential function is a function written in the form $f(x) = a \cdot b^x$. To solve an exponential function, you can use what you know about the properties of exponents and common bases.

To solve the exponential equation $\frac{1}{32} = 2^{x-1}$, first determine the power of 2 that gives the result of 32:

$$(2)(2)(2)(2)(2) = 32$$
$$2^5 = 32$$

Then rewrite the equation to show common bases:

$$\frac{1}{32} = 2^{x-1}$$
$$\frac{1}{2^5} = 2^{x-1}$$
$$2^{-5} = 2^{x-1}$$

Because the expressions on both sides of the equals sign have the same base, you can set up and solve an equation using the exponents:

$$-5 = x - 1$$
$$-5 + 1 = x - 1 + 1$$
$$-4 = x$$

Therefore, $x = -4$.

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Solve each equation for x.

1. $3^x = 81$
2. $2^{4x} = 1$
3. $4^{8-x} = \frac{1}{64}$

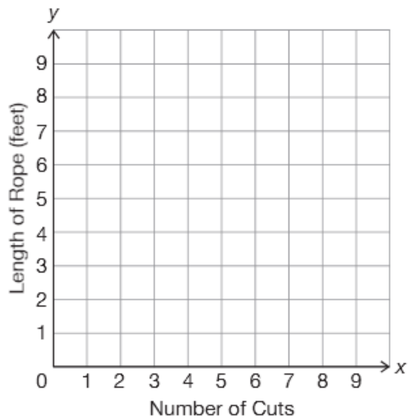
4. $5^{9x} = 5$
5. $\frac{1}{3^{x+5}} = 243$
6. $2^{-x} = \frac{1}{2}$

PROBLEM 3 Inverses

1. The Amazing Aloysius is practicing one of his tricks. As part of the trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 10-foot rope and then cuts it in half. He takes one of the halves and cuts that piece in half. He keeps cutting the pieces in half until he is left with a piece so small he can't cut it anymore.

   a. Complete the table to show the length of rope after each of Aloysius's cuts. Write each length as a whole number, mixed number, or fraction. Then graph the points from the table.

Number of Cuts	Length of Rope (feet)
0	10
1	
2	
3	
4	
c	



   b. Determine which function family represents this situation. Then, write the length of the rope as a function of the cut number.

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c. Use your function to determine the length of the rope after the 7^{th} cut.

2. Write an exponential function of the form $f(x) = ab^x$ given each pair of ordered pairs.

a. $(0, 4)$ and $\left(3, \frac{1}{2}\right)$

b. $\left(-2, -\frac{1}{8}\right)$ and $(0, -2)$

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Be prepared to share your solutions and methods.